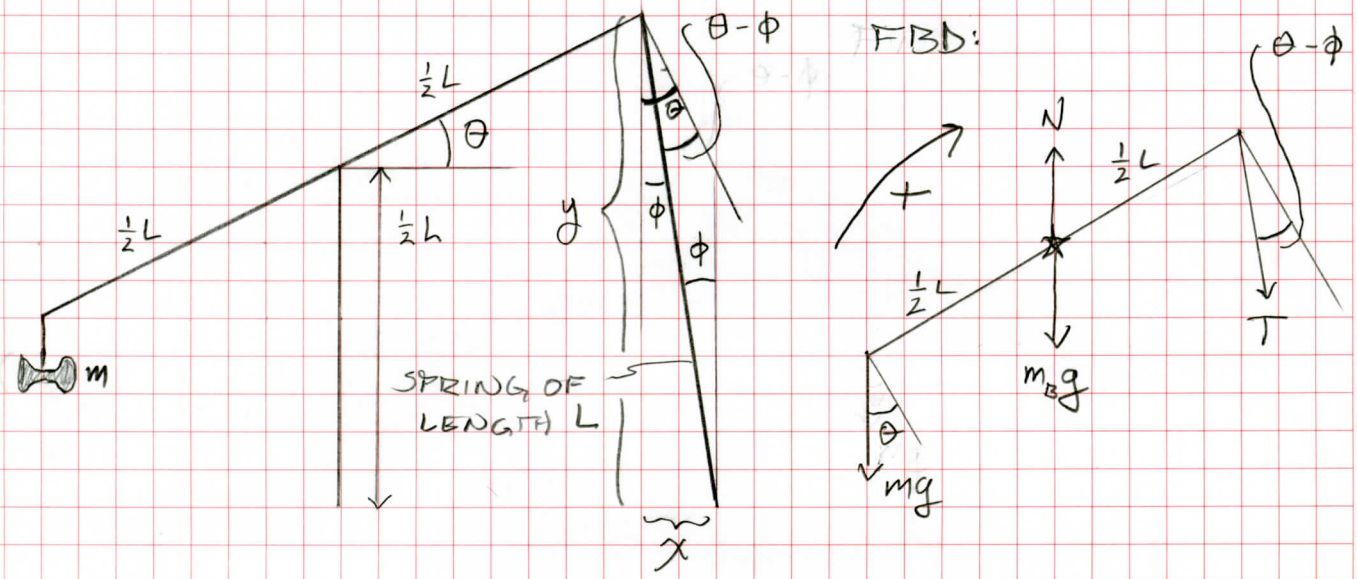
T6 PR 12.59

a) FOR THE BEAM SHOWN, SHOW THAT

$$m = \frac{\frac{1}{2}kL\left(\sqrt{3+2(\sin\theta-\cos\theta)}-1\right)\cos\left[\theta-\tan^{-1}\left(\frac{1-\cos\theta}{1+\cos\theta}\right)\right]}{g\cos\theta}$$

b) EVALUATE FOR $L = 2.5\,m$, $k = 1250\,N/m$ AND $\theta = 17.5^\circ$



APPLY NSL TO TORQUES ABOUT A $\quad \Sigma \tilde{\tau}_A = I\ddot{\phi}^{0}$

$$\left(\tfrac{1}{2}L\right)\left[T\cos(\theta-\phi)\right] - \left(\tfrac{1}{2}L\right)(mg\cos\theta) = 0$$

$$\Rightarrow \boxed{m = \frac{T}{g}\,\frac{\cos(\theta-\phi)}{\cos\theta}} \leftarrow \text{MASS}$$

FIND T DUE TO THE EXTENSION OF THE SPRING

$$L_0 = \tfrac{1}{2}L$$

$$L = \sqrt{x^2+y^2}$$

$$\begin{cases} y = \frac{1}{2}L + \frac{1}{2}L\sin\theta \\ x = \frac{1}{2}L - \frac{1}{2}L\cos\theta \end{cases}$$

$$\Rightarrow L = \sqrt{\tfrac{1}{4}L^2(1-\cos\theta)^2 + \tfrac{1}{4}L^2(1+\sin\theta)^2}$$

$$L = \tfrac{1}{2}L\sqrt{1-2\cos\theta+\cos^2\theta+1+2\sin\theta+\sin^2\theta}$$

$$\cos^2\theta+\sin^2\theta = 1$$

$$L = \tfrac{1}{2}L\sqrt{3+2(\sin\theta-\cos\theta)}$$

$$\Rightarrow T = k\Delta L = \boxed{\tfrac{1}{2}kL\left(\sqrt{3+2(\sin\theta-\cos\theta)}-1\right) = T} \leftarrow \text{SPRING FORCE}$$

T6 PR 12.59 CONTINUED

FIND AN EXPRESSION FOR ϕ

$$\tan\phi = \frac{x}{y} = \frac{\frac{1}{2}L(1-\cos\theta)}{\frac{1}{2}L(1+\sin\theta)} = \frac{1-\cos\theta}{1-\sin\theta} = \tan\phi$$

SO PUT THIS & T INTO THE EQUATION FOR m

$$\boxed{m = \frac{\frac{1}{2}kL\left(\sqrt{3+2(\sin\theta-\cos\theta)}-1\right)\cos\left[\theta-\tan^{-1}\left(\frac{1-\cos\theta}{1+\sin\theta}\right)\right]}{g\cos\theta}}$$

QED!

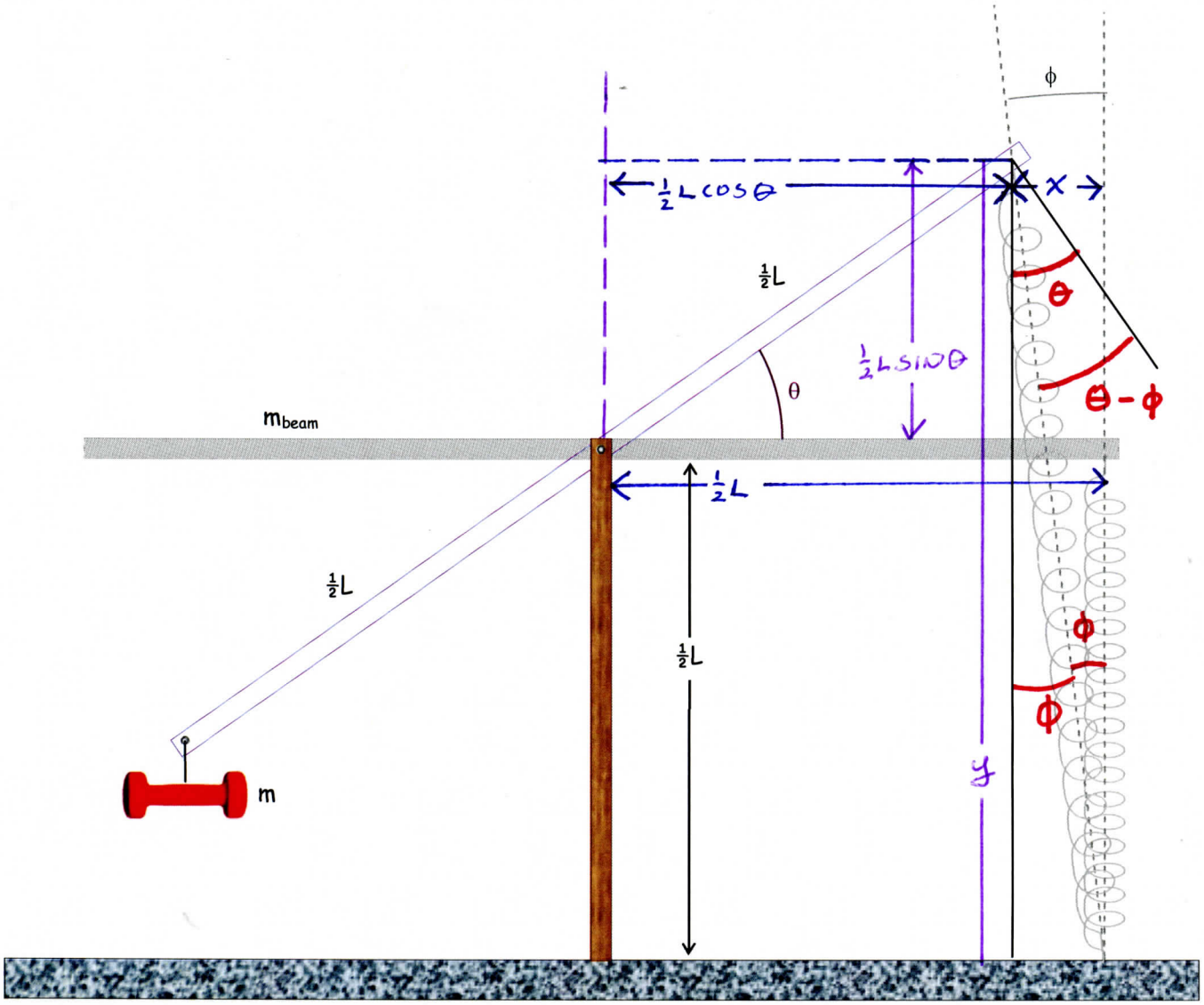
b) EVALUATE

$$m = \frac{\frac{1}{2}(1250)(2.5)\left(\sqrt{3+2(\sin 17.5-\cos 17.5)}-1\right)\cos\left[17.5-\tan^{-1}\left(\frac{1-\cos 17.5^\circ}{1+\sin 17.5^\circ}\right)\right]}{(9.8)(\cos 17.5^\circ)}$$

$$m = \frac{(1562.5)(1.30-1)\cos[17.5-\tan^{-1}(0.0237)]}{9.35}$$

$$m = \frac{471.1\cos[17.5-2.04]}{9.35} \approx \boxed{48.6\ \text{kg} = m}$$ WHEW!

Expanded diagram for T6 Pr. 12-59



$y = \frac{1}{2}L + \frac{1}{2}L\sin\theta$

$x = \frac{1}{2}L - \frac{1}{2}L\cos\theta$